

## COMPUTER-AIDED OPTIMAL METHOD FOR LUBRICATING OIL MIXTURES

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**Abstract:** The industrial process for obtaining commercial lubricating oils has an intrinsic complexity and requires a very special care of the products quality. This is the reason why any improvement in this field is welcome.

A possible way to lower the production cost is to use “optimal mixing recipes”, for getting commercial lubricating oils by the base oils (and additives). This work proposes an original software tool designed to offer these recipes when the final mixture properties are explicitly given. The application runs under MATLAB environment residence, being built-up around a nonlinear programming routine. Its features as robustness, accuracy and good functionality were proved not only by theory, but also in practice, after laboratory experimental tests.

**Keywords:** lubricating oils, mixing recipe, nonlinear programming algorithm.

### 1. INTRODUCTION

The progress in the area of industrial production, agriculture, transports, correlated with the primary resources and energy saving necessity, determined the production of the lubricating oils having a continuously improved quality.

In the industrial plants, obtaining a high quality lubricating oils needs an advanced technology, taking into account the quality specifications and the process complexity.

Generally, the lubricating oils are obtained by combining base oils and additives. On their turn, the base oils are obtained from selected crude oils, through vacuum distillation, solvent extraction, hydro-cracking, catalytic or solvent-based de-waxing and hydrogen or chemicals special treatments (Tanasescu, 2002).

The entire process until obtaining a commercial lubricating oil may be anytime the subject for improvements, from small changes to radical

optimizations, in a particular place or from the plantwide point of view.

This paper deals with a necessary optimization in obtaining the base oils mixtures with imposed properties, in the sense of getting a minimal production cost for a commercial lubricating oil volumetric unity. First, the required mathematical model is presented, then the software application structure is detailed. After the model experimental validation (by laboratory tests), at the final some results (“optimal mixing recipes”) are also presented.

### 2. LUBRICATING OILS – A SHORT CHARACTERIZATION

Between the general tasks that the lubricating oils must accomplish, the most important function is to lubricate the moving surfaces in order to lower the rub effects.

The lubricating oils general tasks, and so the basic task, depend on the physical and chemical properties (viscosity, viscosity index, pour point, flash point, refractive index, acidity, oxidation durability, density, Conradson Carbon Residue, ash content, color, Watson Characterization Factor).

Generally, the base oils are characterized by only three properties (viscosity, pour point, flash point) because among the 23 analytical oil properties, only these are mutual independent.

Taking into account that the lubricating oils may be obtained by mixing base oils, it is necessary to formulate appropriate additivity equations for each characteristic in order to be able to estimate the mixture correspondent characteristic.

Related to some properties, the lubricating oils may be considered as liquid hydrocarbon ideal mixtures, for which properties could be calculated by using linear additivity equations. But in other situations (by example for the properties chosen in this work to characterize a base oil – viscosity, pour point, flash point), the characteristics are non-additive in correlation with the components volumetric fractions. As a consequence, the quoted literature recommends the use of a few transforming equations (Apostol, 1988).

Thus, if the  $P_i$  is the property of the “i” component and  $X_i$  is its volumetric fraction in the mixture, an additive property satisfies the linear equation

$$P_{\text{mix}} = \sum_{i=1}^{nc} P_i \cdot X_i, \quad (1)$$

where  $nc$  is the total number of components in the mixture and  $P_{\text{mix}}$  is the corresponding mixture property.

If  $P_{\text{mix}} \neq \sum_{i=1}^{nc} P_i \cdot X_i$ , the property is non additive and, to keep a linear form of the mixing equation, the  $P$  property has to be replaced by a

transformed variable called “mixing index”,  $MI = f(P)$ , that still satisfies the linear equation

$$MI = \sum_{i=1}^{nc} MI_i \cdot X_i, \quad (2)$$

where  $MI_i$  is the mixing index for the “i” component and  $MI$  is the mixing index for the mixture (Radulescu and Ilea, 1982; Radulescu, 1984).

To be concrete, the transforming equations for viscosity, pour point, flash point, which permit the mixture properties estimation in correlation with the components properties are as follows:

**For viscosity (v):**

$$MI_{-v_i} = 33,46345 \cdot \log \log(v_i + 0,8) + 23,10219; \quad (3)$$

$$MI_{-v_{\text{mix}}} = \sum_{i=1}^{nc} (MI_{-v_i} \cdot X_i); \quad (4)$$

$$v_{\text{mix}} = 10 \exp 10 \exp \left( \frac{MI_{-v_{\text{mix}}} - 23,10219}{33,46345} \right) - 0,8, \quad (5)$$

where  $MI_{-v_i}$  is the mixing index for the “i” component viscosity;

$MI_{-v_{\text{mix}}}$  – the mixing index for the mixture viscosity;

$v_i$  – the viscosity of the “i” component [cSt];

$v_{\text{mix}}$  – the mixture viscosity [cSt].

**For flash point (T):**

$$MI_{-T_i} = 10^{\frac{-T_i}{58,6}}; \quad (6)$$

$$MI_{-T_{\text{mix}}} = \sum_{i=1}^{nc} (MI_{-T_i} \cdot X_i); \quad (7)$$

$$T_{\text{mix}} = -58,6 \cdot \log(MI_{-T_{\text{mix}}}), \quad (8)$$

where  $MI_{-T_i}$  is the mixing index for the “i” component flash point;

$MI_{-T_{\text{mix}}}$  – the mixing index for the mixture flash point;

$T_i$  – the flash point, [ $^{\circ}\text{C}$ ];

$T_{\text{mix}}$  – the mixture flash point, [ $^{\circ}\text{C}$ ].

**For pour point (Pp):**

$$MI\_Pp_i = 10 \exp(9,8645 \cdot 10^{-8} \cdot t_{50\%i}^2 + 1,8782 \cdot 10^{-5} \cdot t_{50\%i} + 0,020396)(Pp_i - 29,276) + 2,3698; \quad (9)$$

$$MI\_Pp_{mix} = \sum_{i=1}^{nc} (MI\_Pp_i \cdot X_i); \quad (10)$$

$$Pp_{mix} = \frac{\log(MI\_Pp_{mix}) - 2,3698}{9,8645 \cdot 10^{-8} \cdot t_{50\%i}^2 + 1,8782 \cdot 10^{-5} \cdot t_{50\%i} + 0,020396} + 29,276, \quad (11)$$

where  $MI\_Pp_i$  is the mixing index for the “i” component pour point;

$MI\_Pp_{mix}$  – the mixing index for the mixture pour point;

$t_{50\%i}$  – the temperature on the “i” component TBP curve corresponding on 50% distilled;

$Pp_i$  – the pour point of the “i” component, [°C];

$Pp_{mix}$  – the pour point of the mixture, [°C].

Obviously, these equations have to be validated by experimental tests. As seen in the following chapter, three base oils were used (a light oil, a medium oil and a heavy oil) for which characteristics were also determined by laboratory experiments. Different mixtures were obtained (by using various volumetric fractions of each component), and, for each resulting mixture, the viscosity, flash point and pour point were experimentally determined. The results presented in the 3<sup>rd</sup> chapter of this work prove the validity and accuracy of the equation from (3) to (11) (Apostol, 1988).

### 3. AN OPTIMAL METHOD FOR LUBRICATING OIL MIXTURES

As shown above, when the physical properties of the base lubricating oils are known, it is possible to estimate in an accurate way the physical properties of a coherent mixture (supposing given the volumetric fractions of each component). On the other way, that means a particular lubricating oil, with desired properties, may be obtained by mixing the basic components, this being the principle of the industrial method.

But, as demonstrated by the theory and confirmed in practice, there are many possible combinations (different fractions) between the basic components, leading to the same imposed properties for the resulting mixture. This way, if an optimal criteria is identified, the method may increase in efficiency by satisfying this criteria (Curievici, 1980).

This paper proposes an original computer-aided optimal method for obtaining a mixture of base lubricating oils with given properties. The chosen optimal criteria is to minimize the cost for obtaining the mixture, so the objective function that should be minimized may be written as:

$$f_{obj}(X_1, X_2, \dots, X_{nc}) = \sum_{i=1}^{nc} (p_i \cdot X_i), \quad (12)$$

where  $p_i$  are the components unitary prices.

The software application developed by authors is written in MATLAB (The MathWorks, Inc., 2000) and may be described by a functional diagram as shown in figure 1, which emphasizes the three programs modules.

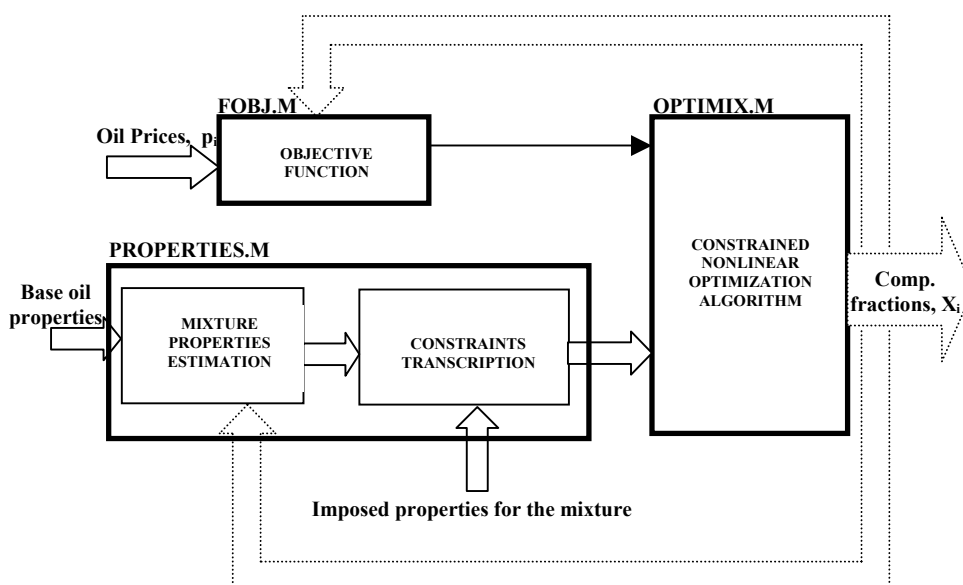


Fig. 1. The optimization application structure diagram.

OPTIMIX.M is the main application module, describing the optimal problem. It makes use of the *fmincon* MATLAB optimizing routine, which finds the minimum of a constrained nonlinear multivariable function,  $\min_x f(x)$ , subject to linear and nonlinear constraints, both equalities and inequalities. This module does not need any care from the user part, it being describing a standard nonlinear programming problem. However, if the algorithm systematically fails to provide pertinent results (according with the physical sense), it is possible to modify the initial optimal point estimation – in this case the base lubricating oils volumetric fractions.

The *fmincon* algorithm needs the other two modules of the application. First, the objective function is implemented in the FOBJ.M file, according with the equation (12). This module is fully customizable, the user being able to change the objective function parameters (the base lubricating oil unitary prices in this case) or even the objective function itself.

Then, the algorithm requires a constraints description, which is implemented in the PROPERTIES.M module. As inputs it needs the physical properties of the base lubricating oils (viscosity, flash point, pour point, TBP curves)

and the imposed physical properties of the resulting mixture, in fact the upper and lower accepted limits for the same characteristics. Basically, this module uses the equations from (3) to (11) to estimate the mixture properties and, having given their limits, transcribes these constraints in a standard form accepted by the *fmincon* routine.

PROPERTIES.M is also fully customizable and, taking into account the basic components and resulting mixture properties are implied, this module initialization is the critical point when a particular application should be solved.

It is not the place here to present all the aspects and problems that could appear when running the optimization program, but as the basic guidelines, the user must take care of the base oils properties accuracy, constraints formulation pertinence and the significance of the initial point for the optimization algorithm.

To give a short overview on the application, the authors present the example of a concrete situation, by determining the optimal volumetric fractions of three base lubricating oils to get a mixture with given properties.

The basic components may be characterized by the data from the table 1.

Table 1. Base oils characteristics

Characteristics	Light oil (LO)	Medium oil (MO)	Heavy oil (HO)
Viscosity at 40°C, cSt	25,57	34,27	125,48
Viscosity at 100°C, cSt	4,88	5,6	12,7
Viscosity index	114	101	92
Pour point, °C	-9	-4	-6
Flash point, °C	214	224	241
t <sub>50%</sub> , °C	445	480	530
Refractive index	1,14849	1,48650	1,49090
Unitary prices, \$	230	240	250

To test the equations from (3) to (11) relevance, different volumetric fractions of base lubricating oils were mixed to make a comparison between the experimental results (laboratory tests for determining the mixture viscosity, flash point and

pour point) and the results given by the PROPERTIES.M module. As shown in the tables 2, 3 and 4, there is a very good concordance between the experiments and the software application properties estimation.

Table 2. Mixture characteristics (viscosity)

Mixture %LO+%MO+%HO	MI_v <sub>mix</sub>	Experimental viscosity, cSt	Calculated viscosity, cSt
10+30+60	31,98	66,70	68,80
20+20+60	31,86	65,20	66,40
30+20+50	31,29	53,93	56,40
50+30+20	29,70	35,80	36,80
30+50+20	29,95	39,00	39,30
80+10+10	28,89	28,80	30,10
20+60+20	30,07	39,00	40,50

Table 3. Mixture characteristics (pour point)

Mixture %LO+%MO+%HO	t <sub>50%</sub> , °C	MI_Pp <sub>mix</sub>	Experimental pour point, °C	Calculated pour point, °C
10+30+60	506	2,879	-5,0	-5,6
20+20+60	503	2,774	-6,0	-6,0
30+20+50	494	2,892	-6,0	-6,3
50+30+20	472	3,348	-7,0	-6,6
30+50+20	479	3,557	-5,0	-5,6
80+10+10	457	3,256	-10,0	-8,0
20+60+20	483	3,661	-5,0	-5,2

Table 4. Mixture characteristics (flash point)

Mixture %LO+%MO+%HO	MI_T <sub>mix</sub>	Experimental flash point, °C	Calculated flash point, °C
10+30+60	0,000113	234	231
20+20+60	0,000121	230	229
30+20+50	0,000135	228	226
50+30+20	0,000171	220	220
30+50+20	0,000157	225	222
80+10+10	0,000200	220	216
20+60+20	0,000150	225	224

The above results prove that in this particular case, the PROPERTIES.M module gives significant information, so it is appropriate to be used in the optimization problem.

To test the entire application, different properties for the mixture were imposed. In each case, the

nonlinear programming algorithm gave the optimal volumetric fractions for each component, as well as the objective function value (the minimal unitary cost), as seen in the table 5.

Table 5. Optimal mixing recipes.

Imposed limits (min/max) for the mixture properties			% optimal			Calculated mixture properties			Minimum unitary cost, \$
v, cSt	Pp, °C	T, °C	LO	MO	HO	v, cSt	Pp, °C	T, °C	
58 / 62	-6 / -8	226 / 230	24,0	22,0	54,0	60,30	-7	228	242,97
58 / 62	-6 / -8	224 / 228	42,0	0,0	58,0	60,00	-8	226	241,62
50 / 54	-8 / -10	224 / 228	44,6	7,3	48,1	52,40	-8	225	240,35
30 / 35	-6 / -10	215 / 220	61,0	22,0	17,0	34,40	-8	219	235,60
32 / 36	-6 / -10	218 / 222	57,7	25,0	17,3	34,90	-8	220	235,97
28 / 32	-6 / -10	215 / 220	80,0	10,0	10,0	30,15	-9	217	233,00
33 / 37	-6 / -8	218 / 225	39,0	47,6	13,4	35,30	-6	221	237,40

#### 4. CONCLUDING REMARKS

The technology used to obtain commercial lubricating oils, due to its complexity, may be subject for improvements, from small changes to radical optimization.

This paper presented a software application developed by authors which helps in getting “the optimal mixing recipes” for obtaining lubricating oils from base oils, the optimal criteria being the production cost minimization. The necessary mathematical model was presented (proving its validity by laboratory tests), then the software application functional structure was detailed. Just as example, at the final some “optimal recipes” for getting mixtures with imposed properties are also presented.

The proved application good functionality and robustness make it a necessary tool in any

laboratory, but also for the industrial process assistance.

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